Chapter 2
Sections 2.4 & 2.5

Problem 1. Let $f(x) = 3x - 1$ and $g(x) = 1 - x^2$. Find the following.

(a) $g(f(2))$ $f(2) = 6 - 1 = 5$

$g(f(2)) = g(5) = 1 - 25 = -24$

(b) $g(f(x))$ $g(3x-1) = 1 - (3x-1)^2 = 1 - 9x^2 + 6x - 1 = -9x^2 + 6x$

(c) $f(g(x))$ $f(1-x^2) = 3 - 3x^2 - 1 = 2 - 3x^2$

(d) $f(f(x))$ $f(3x-1) = 9x - 3 - 1 = 9x - 4$

Problem 2. A banner printing company charges $C(a) = 5a + 25$ dollars to print a banner with an area of a square feet. Aja wants to print a banner that is 2 feet wide. If her banner is x feet long, the area of her banner is given by $A(x) = 2x$.

(a) Evaluate and interpret $C(A(10))$ in the context of the problem.

$A(10) = 20$ $C(20) = 125$ the cost of printing a banner 2 ft height and 10 ft long

(b) What are the units of the inputs and the outputs of the function $C(A(x))$?

input → ft output → cost

Problem 3. The height, H, in feet, of water in a cylindrical tank is given by $H(t) = 0.25t$, where t is time in hours. The volume of water in the tank when the height is h feet is $V(h) = 25\pi h$ cubic feet. Find a formula for $V = f(t)$, the volume of water in the tank after t hours.

$S(t) = V(H(t)) = 25\pi(0.25)t$

Problem 4. Composing multiple functions becomes challenging when considering domain and range. Consider $f(x) = \frac{1}{x-3}$ and $g(x) = \sqrt{x}$ and answer the following questions.

(a) Find the domain and range of $f(x)$.

D: $x \neq 3$ range: $y \neq 0$

(b) Find the domain and range of $g(x)$.

D: $x \geq 0$ R: $y \geq 0$

(c) Write out $f(g(x))$ and find the domain and range of $f(g(x))$.

$\frac{1}{\sqrt{x}-3}$ D: $x \neq 9$ & $x \geq 0$ R: $y \neq 0$

(d) Write out $g(f(x))$ and find the domain and range of $g(f(x))$.

$\frac{1}{\sqrt{x-3}}$ D: $x > 3$ R: $y > 0$

(e) What is the relationship between the domain of $f(x)$ and the domain of $g(f(x))$?

(f) What is the relationship between the domain of $g(x)$ and the domain of $f(g(x))$?

great Bonus Question!

Problem 5. Fill in the blanks using the table below.

x	-10	-7	-4	-1	2
$g(x)$	1	13	-3	7	11

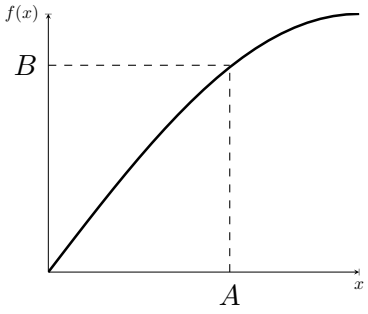
(a) $g(-7) =$ 13

(b) $g($ 2 $) = 11$

(c) $g^{-1}($ 13 $) = -7$

(d) $g^{-1}(11) =$ 2

Problem 6. Using the graph of $f(x)$ below, fill in the blanks:



(a) $f(0) =$ D

(b) $f($ A $) = B$

(c) $f^{-1}(B) =$ A

(d) $f^{-1}($ D $) = 0$

Problem 7. USC student Madison is studying abroad in Copenhagen, Denmark.

(a) She knows that the temperature in degrees Fahrenheit, F, is given by the equation

$$F = g(C) = \frac{9}{5}C + 32,$$

where C is the temperature in degrees Celsius.

(i) Find a formula for $g^{-1}(F)$, and interpret its meaning in the context of the problem. What are the units of the inputs and outputs?

$g^{-1}(F) = \frac{5(F-32)}{9}$ function that gives degrees in celcius if given degrees farhenheit

(ii) Evaluate and interpret $g(0)$ and $g^{-1}(0)$.

$g(0) = 32$ $g^{-1}(0) = \frac{(-32)\cdot 5}{9}$

32 Farhenheit is 0 degrees celcius ...

(b) Madison also always needs to keep the currency exchange rate in mind: $D = f(K) = 0.15K$, where D is the value in US dollars, and K is the value in Danish krones.

(i) Find a formula for $f^{-1}(D)$, and interpret its meaning in the context of the problem. What are the units of the inputs and outputs?

$f^{-1}(D) = \frac{D}{0.15}$ any D-dollars is how

(ii) Evaluate and interpret $f(60)$ and $f^{-1}(90)$.

$f(60) = (0.15)(60)$ many krones is equal to 60

$f^{-1}(90) = \frac{90}{0.15}$ dollars equal to 90 krones

(c) Suppose the monthly cost in Danish krones of heating Madison's apartment to x degrees Celsius is given by $H(x) = 32x$. Evaluate and interpret $f(H(g^{-1}(70))$.

$0.15 \cdot \left(32 \cdot \left(\frac{5(F-32)}{9}\right)\right)$

Problem 8. *Synthesis Problem.* A team of American engineers is working with a team in Germany to design an engine. They need to know the volume of the interior of a piston; in this engine, the volume of each piston is given by $V(x) = 10.125\pi x^2$ where x is the diameter of the piston in centimeters. However, the American team took all their measurements in inches. There are 2.54 centimeters in each inch. Write a formula that converts inches to centimeters, and then give a formula for the volume of the cylinder which has input in inches.

$f(y) = 2.54 \cdot y$

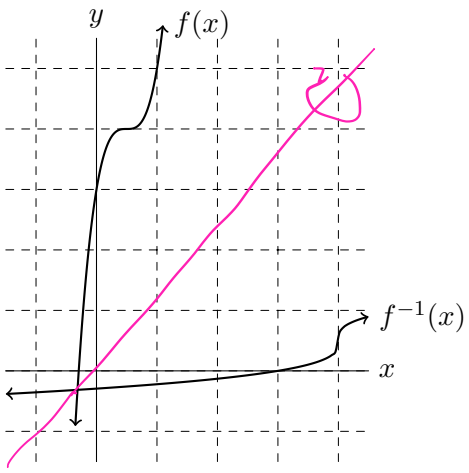
$V(f(y)) = 10.125\pi (2.54y)^2$

Problem 9. Let $f(x) = (2x-1)^3 + 4$.

(a) Find $f^{-1}(x)$.

$f^{-1}(x) = \frac{\sqrt[3]{x-4} + 1}{4}$

(b) Consider the graphs of $f(x)$ and $f^{-1}(x)$ below. What is their graphical relationship?



Problem 10. A lot of people think $f(x) = x^2$ and $g(x) = \sqrt{x}$ are inverse functions. But notice that $g(f(-2)) = g(4) = \sqrt{4} = +2$. So $g(f(-2)) = 2$. What went wrong? Can we restrict the domain so it does work?

another great Bonus